whence, $\sin\alpha = 2bc\sqrt{[s(s-1/a)(s-1/b)(s-1/c)]}$;

$$x = a\sin\alpha = 2abc\sqrt{[s(s-1/a)(s-1/b)(s-1/c)]}$$

$$=\frac{1}{2abc}\sqrt{[(bc+ac+ab)(-bc+ac+ab)(bc-ac+ab)(bc+ac-ab)]}.$$

Solved similarly by W. S. Risley, H. C. Feemster, A. M. Harding, G. W, Hartwell, Elmer Schuyler, and A. H. Holmes.

GEOMETRY.

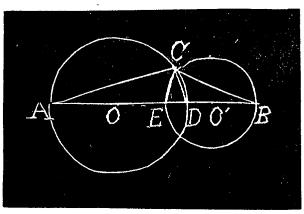
403. Proposed by C. E. GITHENS, Wheeling, West Virginia.

In a triangular field the sides enclosing an obtuse angle are 35 rods and 48 rods in length, Two straight lines are drawn from this vertex, and are at right angles to these sides. If these lines intersect the base 16 rods apart, how long is the third side of the field?

Solution by J. SCHETFER, A. M., Hagerstown, Maryland.

Let there be two intersecting circles; AB the line of centers, C one point of intersection. Draw AC, BC, CD, CE; put $AC=b$, $BC=a$, $DE=m$, radius of circles O and O', respectively, R and r.

In triangles ACE and BDC, respectively, we have $2R-m : b = \cos(A+B) : \cos B$; $2r-m : a = \cos(A+B) : \cos A$. Eliminating $\cos(A+B)$ and considering $2R\cos A = b$, $2r\cos B = a$, we get $a^2R(2R-m) = b^2r(2r-m)$ .. (1).



Since $\sin A : \sin B = a : b$, we have $b^2(1-\cos^2A) = a^2(1-\cos^2B)$; therefore, $b^2r^2(4R^2-b^2) = a^2R^2(4r^2-a^2)$.

From (2) we get

$r^2 = \dfrac{a^4R^2}{4R^2(a^2-b^2)+b^4}$ ...(3). Substituting in (1) and making all necessary reductions, involving merely simple algebraic operations, we get

$$16(a^2-b^2)(R^5-16m(a^2-b^2)R^4-4(a^2-b^2)(2b^2-m^2)R^3+4a^2b^2mR^2$$
$$+b^4(a^2-b\ \ -2m^2)R-b^6m=0,$$

or, divided by $16(a^2-b\ \ )$,

$$R^5-mR^4-\frac{2b^2-m^2}{4}R^3+\frac{a^2b^2m}{4(a^2-b^2)}R^2+\frac{b^4(a^2-b^2-2m^2)}{16(a^2-b^2)}R-\frac{b^6m}{16(a^2-b^2)}=0.$$

This equation of the fifth degree is to be solved for R, then r is found from (3), and $AB=2(R+r)-m$. For $a=35$, $b=48$, $m=16$, the equation is

$$R^5-16R^4-1088R^3-10463.024R^2+488494.5467R+6415370.2=0.$$